Lab Guide

This guide applies to all three labs. Labs 1 and 2 are **formative**: they prepare you, and the measurements you make in them, for the final **summative** Signals Lab, which is assessed by a lab report.

L0.1 General instructions

Bring a notebook to every lab and keep careful, organized records of all measured quantities, settings and error estimates. No measurement is meaningful without an estimate of its error! Keep detailed notes of any computed quantities and of the qualitative observations you make during the experiment: they may be important during your analysis.

Each lab handout contains grey **data-entry boxes**. Fill them in on the printed handout (*Download lab PDF* on the website) as you go.

L0.2 Error propagation

When you calculate with measured quantities that have uncertainties (errors), you need to propagate these errors through the calculation. Suppose two random variables A and B have standard deviations (errors) σ_A and σ_B. The error in a function of these variables is then calculated as follows.

Addition and subtraction

If $C = A \pm B$, the error in C is

$$\sigma_C = \sqrt{\sigma_A^2 + \sigma_B^2} \tag{L0.1}$$

This formula applies to both addition and subtraction: errors *always add in quadrature*, even for subtraction.

Multiplication and division

If $C = A \times B$ or $C = A/B$, the *relative* error in C is

$$\frac{\sigma_C}{C} = \sqrt{\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2} \tag{L0.2}$$

To find the absolute error σ_C, multiply both sides by C:

$$\sigma_C = |C|\sqrt{\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2}.$$

Example

Suppose we measure $R_R = (100.5 \pm 0.5)\,\Omega$ and $R_L = (12.3 \pm 0.3)\,\Omega$, and we want the total resistance $R = R_R + R_L + R_{\text{int}}$, where $R_{\text{int}} = (600 \pm 50)\,\Omega$. Using the addition rule (L0.1),

$$\begin{aligned} R &= 100.5 + 12.3 + 600 = 712.8\,\Omega, \\ \sigma_R &= \sqrt{0.5^2 + 0.3^2 + 50^2} = \sqrt{0.25 + 0.09 + 2500} \approx 50.0\,\Omega. \end{aligned}$$

Therefore $R = (712.8 \pm 50.0)\,\Omega \approx (710 \pm 50)\,\Omega$ (rounded appropriately).

Lab 1

Resistance and Capacitance

Before the lab, read the Lab Guide (general instructions and error propagation) and the Background section below, and answer Question 1.

L1.1 Goals

- Measure the resistance of resistors in series and in parallel and verify the theoretical predictions.
- Accurately measure the capacitance of a capacitor indirectly, using the growth/decay curves of an RC circuit.
- Familiarize yourselves with the setup and operation of an oscilloscope.
- Familiarize yourselves with the scope's software and analysis tools.
- Familiarize yourselves with the use of breadboards and basic circuit components such as capacitors and resistors.
- (Optional) Measure the capacitance of capacitors in series and in parallel and verify the theoretical predictions.

L1.2 Background

Resistance and capacitance in series and parallel

The effective resistance of resistors in series, R_s, and in parallel, $R_\parallel$, is

$$\begin{aligned} R_s &= R_1 + R_2 + \dots \\ \frac{1}{R_\parallel} &= \frac{1}{R_1} + \frac{1}{R_2} + \dots, \end{aligned} \tag{L1.1}$$

where R_i is the resistance of the i-th resistor. Similarly, for capacitors,

$$\begin{aligned} \frac{1}{C_s} &= \frac{1}{C_1} + \frac{1}{C_2} + \dots \\ C_\parallel &= C_1 + C_2 + \dots. \end{aligned} \tag{L1.2}$$

Measuring capacitance

While resistance can be measured directly using a multimeter, capacitance cannot: $C = Q/V$, and the charge Q on the plates cannot be measured directly. Capacitance can, however, be measured

indirectly by determining the characteristic charge/discharge time $\tau_{\mathrm{RC}} = RC$ of an RC circuit. Knowing R then gives

$$\boxed{C = \frac{\tau_{\mathrm{RC}}}{R}} \tag{L1.3}$$

In class (Current, Resistance & DC Circuits), we found that the voltage across the capacitor of an RC circuit is

$$\begin{aligned} V_{\mathrm{C,charge}}(t) &= V_0 \left(1 - e^{-t/RC}\right) \\ V_{\mathrm{C,discharge}}(t) &= V_0\, e^{-t/RC}. \end{aligned} \tag{L1.4}$$

In this experiment, we will build an RC circuit and use it to measure C indirectly.

To do this, we will use a wave generator to produce a square wave that drives the RC circuit. This alternates the voltage between $-V_0$ and V_0 at regular intervals, so that the capacitor is alternately charged and discharged. We will then use an oscilloscope probe to measure the voltage across the resistor. Since the current through the circuit is $I = C\, dV_{\mathrm{C}}/dt$, the voltage across the resistor is found by taking the derivative of the charging curve in (L1.4) and multiplying by RC:

$$\boxed{V_{\mathrm{R}}(t) = V_0\, e^{-t/RC}} \tag{L1.5}$$

We will use the software provided with the oscilloscope to measure the **fall time**: the time it takes for the voltage to fall from 80% to 20% of its maximum value. Call this time Δt. Note that it does not depend on V_0, so we do not need to know the exact amplitude of the signal delivered to the circuit.

Question 1: Using (L1.5), compute τ_{RC} from the fall time Δt. Insert your answer below.

$$\tau_{\mathrm{RC}} = _____$$

L1.3 Materials

- Multimeter
- Oscilloscope (10 MHz Picoscope 2204A with accompanying software) and two probes
- Wave generator
- Breadboard
- Capacitors (rated 15 nF and 100 nF)
- Resistors (two rated 2 kΩ and one rated 100 kΩ)
- Wires

L1.4 Instructions

Resistors in series and parallel

- Use the multimeter to measure and record the resistance of the two 2 kΩ resistors and of the 100 kΩ resistor. Record the minimum resolution of the meter reading: this gives an estimate of the error in the resistance. Note that the actual resistance may differ from its rated value.

Resistance measurements

First 2 kΩ resistor (R_1): _____ ± _____ kΩ
(measured with multimeter)

Second 2 kΩ resistor (R_2): _____ ± _____ kΩ
(measured with multimeter)

100 kΩ resistor (R_R): _____ ± _____ kΩ
(measured with multimeter)

- Arrange the two 2 kΩ resistors in series and in parallel on the breadboard and measure their effective resistance using the multimeter. Compare the measured values with theory.

Series and parallel resistance

Series resistance (R_s): _____ ± _____ kΩ
(measured with multimeter)

Parallel resistance ($R_\parallel$): _____ ± _____ kΩ
(measured with multimeter)

Question 2: Using (L1.1), predict the series and parallel resistance of the two 2 kΩ resistors. Are the predicted values distinguishable from the rated value of a single resistor, given the resolution of the multimeter?

Measuring capacitance

- Set up the series RC circuit of Figure L1.1 as shown in Figure L1.2. Use the 100 kΩ resistor and the 15 nF capacitor, and note the capacitor's rating.

Capacitor rating

Capacitor rating (C_{rating}): _____ nF

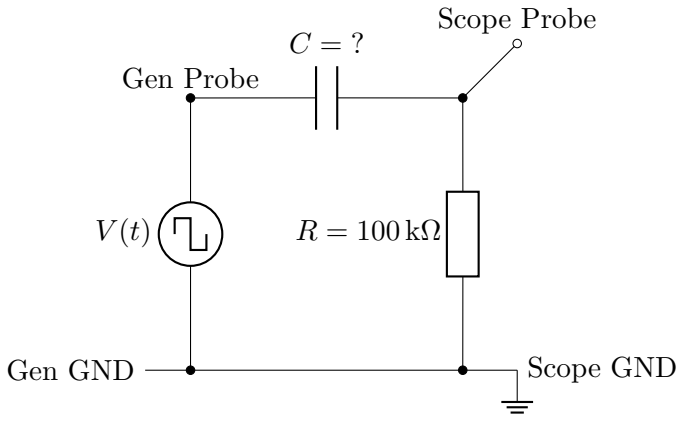


Figure L1.1: RC circuit driven by a square-wave generator. The oscilloscope measures the voltage across the resistor (between Scope Probe and Scope GND).

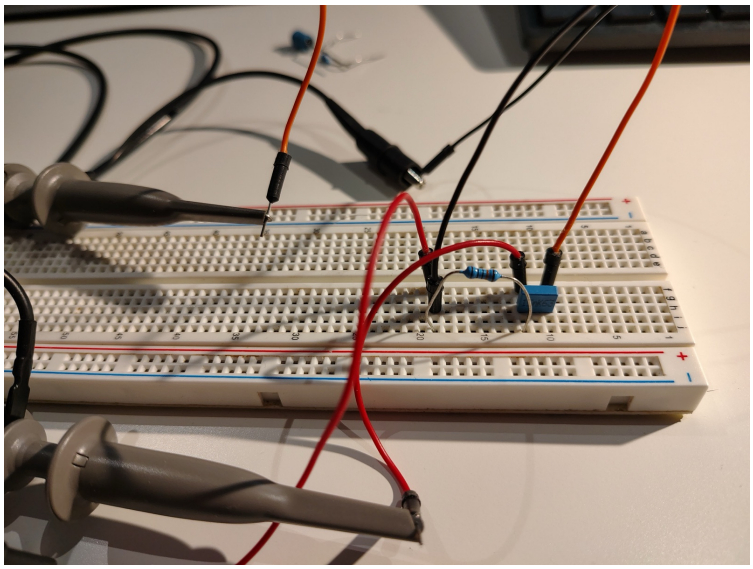


Figure L1.2: The RC circuit of Figure L1.1 on a breadboard.

- Use the measured value of R and the rated value of C to estimate τ_{RC}.

Time constant estimate

$\tau_{\mathrm{RC,\ estimate}}$: _____ ms

- Using the Picoscope software, set the timescale on the scope to 2 ms/div, the voltage scale to a maximum of 2 V, and the trigger to auto (you may later need to adjust the trigger level manually to get a stable signal).
- Set the waveform generator to the square-wave setting, with a frequency between 50 Hz and 70 Hz.

Question 3: Why are these good timescale and frequency settings?

- Once you have a clear signal, use the "measurements" button to measure the fall time over many cycles ($n > 100$). Note the value of σ, the standard deviation of the measurement, which is useful for estimating the random error.

Fall time measurement (15 nF)

Fall time (Δt): _____ ± _____ ms
(from oscilloscope measurements)

Standard deviation (σ): _____ ms

- Repeat the measurement with the 100 nF capacitor. You will need to re-estimate τ_{RC} and change the scope timescale and generator frequency accordingly (see Question 3).

Fall time measurement (100 nF)

Fall time (Δt): _____ ± _____ ms
(from oscilloscope measurements)

Standard deviation (σ): _____ ms

- **(Optional, if time allows.)** Place the two capacitors in series and then in parallel, and repeat the measurement for each combination. Remember to estimate τ_{RC} first to choose reasonable scope and generator settings.

Capacitors in series and parallel

Series fall time (Δt_s): _____ ± _____ ms
(from oscilloscope measurements)

Parallel fall time ($\Delta t_{\|}$): _____ ± _____ ms
(from oscilloscope measurements)

L1.5 Analysis

- Do your results confirm or refute the formulas (L1.1) for resistance in series and in parallel?
- Compute the measured value of C for each capacitor using the measured value of Δt, your answer to Question 1, and (L1.3). Compare this to its rated value.
- Estimate the error in these measurements using σ and your error estimate for R (see Error propagation in the Lab Guide).
- (Optional) Do your results for the capacitors in series and parallel confirm or refute the formulas (L1.2) for capacitance? Compare to the values predicted using your measured values of C.
- What are possible sources of systematic error in this measurement? (Hint: the input resistance of the scope is $1\,\mathrm{M}\Omega$. How does this compare to R, and how does the scope probe connect to the circuit? What about the internal resistance of the wave generator, which is around $600\,\Omega$?)

Keep a record of these results. You will use them in the next lab and in the Signals Lab!