Lab Guide

This guide applies to all three labs. Labs 1 and 2 are **formative**: they prepare you, and the measurements you make in them, for the final **summative** Signals Lab, which is assessed by a lab report.

L0.1 General instructions

Bring a notebook to every lab and keep careful, organized records of all measured quantities, settings and error estimates. No measurement is meaningful without an estimate of its error! Keep detailed notes of any computed quantities and of the qualitative observations you make during the experiment: they may be important during your analysis.

Each lab handout contains grey **data-entry boxes**. Fill them in on the printed handout (*Download lab PDF* on the website) as you go.

L0.2 Error propagation

When you calculate with measured quantities that have uncertainties (errors), you need to propagate these errors through the calculation. Suppose two random variables A and B have standard deviations (errors) σ_A and σ_B. The error in a function of these variables is then calculated as follows.

Addition and subtraction

If $C = A \pm B$, the error in C is

$$\sigma_C = \sqrt{\sigma_A^2 + \sigma_B^2} \tag{L0.1}$$

This formula applies to both addition and subtraction: errors *always add in quadrature*, even for subtraction.

Multiplication and division

If $C = A \times B$ or $C = A/B$, the *relative* error in C is

$$\frac{\sigma_C}{C} = \sqrt{\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2} \tag{L0.2}$$

To find the absolute error σ_C, multiply both sides by C:

$$\sigma_C = |C|\sqrt{\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2}.$$

Example

Suppose we measure $R_R = (100.5 \pm 0.5)\,\Omega$ and $R_L = (12.3 \pm 0.3)\,\Omega$, and we want the total resistance $R = R_R + R_L + R_{\text{int}}$, where $R_{\text{int}} = (600 \pm 50)\,\Omega$. Using the addition rule (L0.1),

$$\begin{aligned} R &= 100.5 + 12.3 + 600 = 712.8\,\Omega, \\ \sigma_R &= \sqrt{0.5^2 + 0.3^2 + 50^2} = \sqrt{0.25 + 0.09 + 2500} \approx 50.0\,\Omega. \end{aligned}$$

Therefore $R = (712.8 \pm 50.0)\,\Omega \approx (710 \pm 50)\,\Omega$ (rounded appropriately).

Lab 3

Signals Lab: RLC Circuits

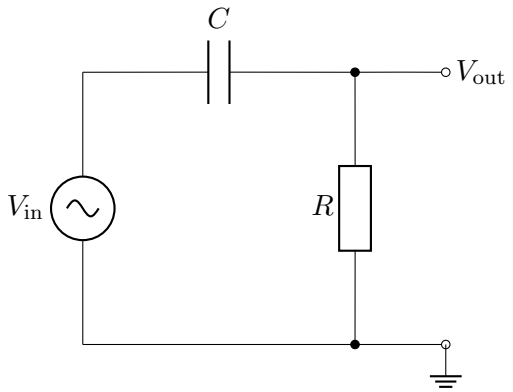
This is the summative lab: it is assessed by a lab report (see the instructions at the end). Before the lab, read the Background section below. The error propagation rules you will need are in the Lab Guide, and you will use your results from Lab 1 and Lab 2.

L3.1 Goals

- Determine the (high-pass) response curve across the resistor of a series RC circuit, and use it to measure the capacitance.
- Determine the (band-pass) response curve across the resistor of a series RLC circuit, and use it to measure the resistance and inductance.
- Determine the ringdown curve of a charged series RLC circuit, and use it to measure the resistance and inductance.
- Learn to estimate and evaluate random and systematic errors in different experimental setups.

L3.2 Background

RC circuits as high-pass filters



As we saw in class (Introduction to Signal Processing), a series RC circuit can function as a high-pass filter when the output is taken across the resistor. At low frequencies the capacitor acts effectively as an open circuit, and at high frequencies as a short circuit, allowing high-frequency signals to pass through to the resistor.

Recall from class that the response function $h(\omega)$, which measures the amplitude attenuation of a pure sine wave with angular frequency ω, is

$$h(\omega) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{RC\omega}{\sqrt{1 + (RC\omega)^2}} \tag{L3.1}$$

where V_{in} and V_{out} are the voltage amplitudes of the incoming and filtered sine waves. The cutoff angular frequency ω_c, defined by $h(\omega_c) = 1/\sqrt{2}$, is

$$\omega_c = \frac{1}{RC}, \qquad \text{or, as a frequency,} \qquad f_c = \frac{1}{2\pi RC}. \tag{L3.2}$$

For frequencies much larger than f_c, $h \approx 1$ and the signal passes through unattenuated. For frequencies much smaller than f_c, the signal drops linearly with frequency: $h(\omega) \approx RC\omega = \omega/\omega_c = f/f_c$. Figure L3.1 shows this behaviour for values of R and C similar (but not identical) to those we will encounter in this lab.

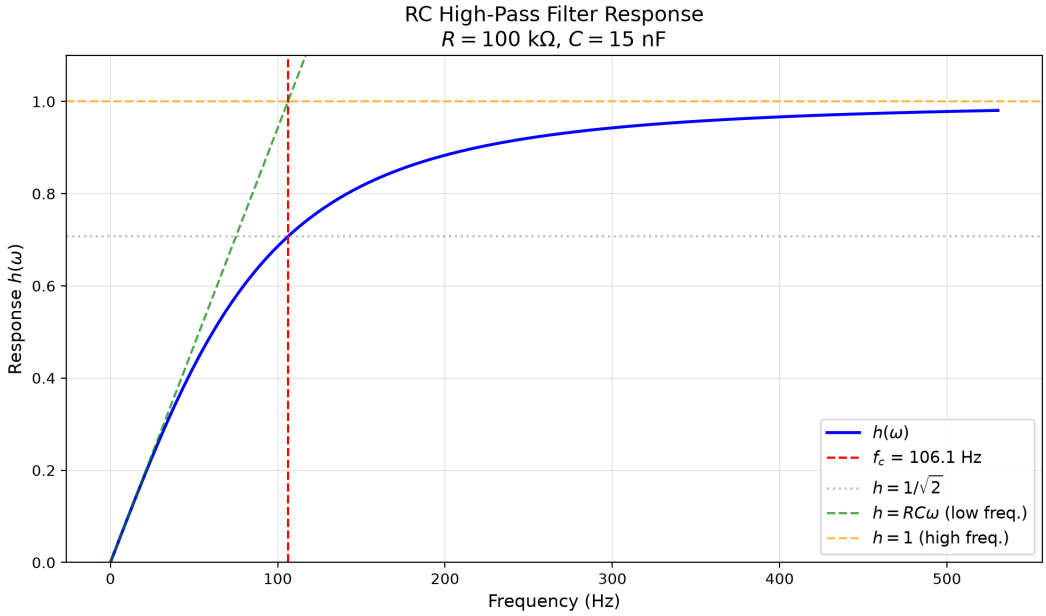
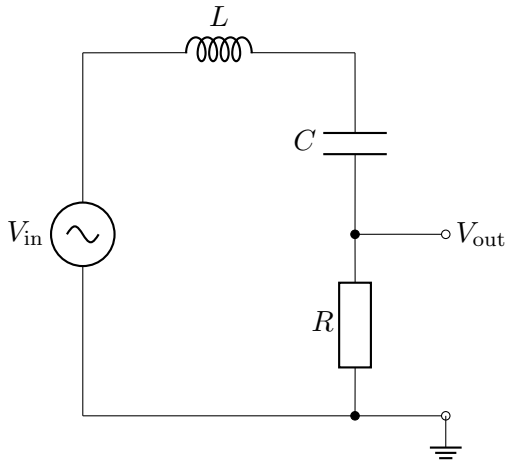


Figure L3.1: Frequency response of the RC high-pass filter, $h(\omega)$ as a function of frequency, with the low-frequency asymptote $h = RC\omega$ and the high-frequency asymptote $h = 1$ for reference.

RLC circuits as band-pass filters



A series RLC circuit can function as a band-pass filter when the output is taken across the resistor. The inductor and capacitor form a resonant circuit that lets signals near the resonance frequency pass, while attenuating both low- and high-frequency signals.

Recall that we derived in class (the resonance curve of the driven RLC circuit) that the frequency response across the resistor of a series RLC circuit is

$$h(\omega) = \frac{1}{\sqrt{\left(\dfrac{\omega^2 - \omega_0^2}{\omega\,\Delta\omega}\right)^2 + 1}} \tag{L3.3}$$

where ω_0 is the **resonant frequency** and $\Delta\omega$ is the **full width at half maximum**, or simply the **bandwidth**:

$$\omega_0 = \frac{1}{\sqrt{LC}}, \qquad \Delta\omega = \frac{R}{L}. \tag{L3.4}$$

The two frequencies at which $h = 1/\sqrt{2}$ are separated by exactly $\Delta\omega$; when $\omega_0 \gg \Delta\omega$ they lie at $\omega_0 \pm \frac{1}{2}\Delta\omega$. The **quality factor**

$$Q = \frac{\omega_0}{\Delta\omega} \tag{L3.5}$$

measures the sharpness of the peak: values much larger than 1 mean sharper peaks. When R, L and C are tuned so that Q is large, the RLC circuit only lets a very narrow band of frequencies pass, and the circuit is said to "resonate" at those frequencies. Figure L3.2 shows this behaviour for parameters similar (but not identical) to those studied in this experiment. Note that the quality factor in this experiment is relatively small, $Q \approx 1.9$, so the passband is relatively wide.

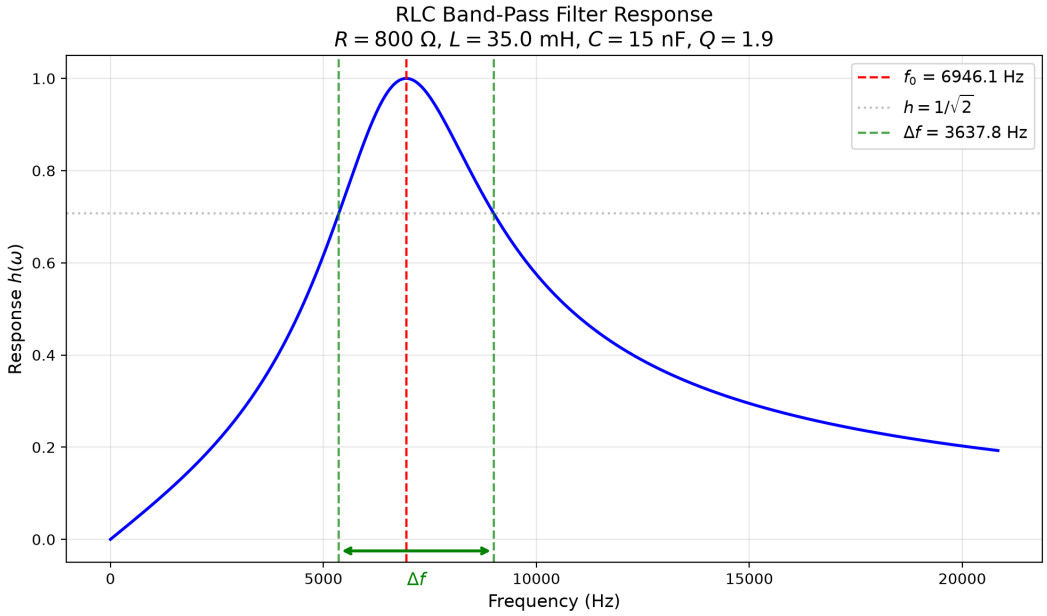


Figure L3.2: Frequency response of the RLC band-pass filter, $h(\omega)$ as a function of frequency, with the resonance frequency f_0 and the bandwidth $\Delta f = \Delta\omega/2\pi$ marked.

Ringdown in an RLC circuit

In class (RL, RLC and AC Circuits), we studied what happens in an RLC circuit if the applied voltage is suddenly dropped to zero: the circuit becomes mathematically equivalent to a damped harmonic oscillator, (5.3). The voltage across the resistor is

$$\boxed{V_{\text{out}}(t) = V_0\, e^{-t/\tau} \sin(\omega' t - \phi)} \tag{L3.6}$$

where the **decay time** τ and the ringdown frequency ω' are

$$\tau = \frac{2L}{R}, \qquad \omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \sqrt{\omega_0^2 - \frac{1}{\tau^2}}. \tag{L3.7}$$

This formula holds provided $\omega_0\tau = 2Q > 1$, which will be satisfied here. The parameters of (L3.6) are V_0, which is determined by the initial voltage, and ϕ, which is determined by the initial rate of change, if any, of the voltage. We will set things up so that $\phi = 0$.

In this experiment, we will use the wave generator to produce a sudden drop in the voltage using a step function, and use the oscilloscope probe to measure the voltage response across the resistor and verify (L3.6). We can measure the period $T = 2\pi/\omega'$ of the oscillations by noting that the sine goes through zero at $t = 0, \frac{T}{2}, T, \ldots$. Since $e^{-t/\tau} > 0$, $V_{\text{out}}(t)$ in (L3.6) can *only* go through zero when the oscillating part does. So you can measure the period by noting where V_{out} goes through zero.

To find τ, we use the fact that the sine is periodic: $\sin(\omega' t) = \sin\big(\omega'(t+T)\big)$. If we choose some time t_1 (say, when the function is at a peak), the oscillating part has the *same* value at $t = t_1 + T$; only the exponential $e^{-t/\tau}$ has changed. Calling $V_1 = V_{\text{out}}(t_1)$ and $V_2 = V_{\text{out}}(t_1 + T)$, we see that

$$\frac{V_1}{V_2} = e^{T/\tau}.$$

Taking logarithms of both sides and rearranging gives

$$\boxed{\tau = \frac{T}{\ln(V_1/V_2)}} \tag{L3.8}$$

So by measuring V_1 and V_2, and using the measured value of T, we can calculate τ. Figure L3.3 illustrates the procedure for values similar (but not identical) to those we will encounter in this experiment.

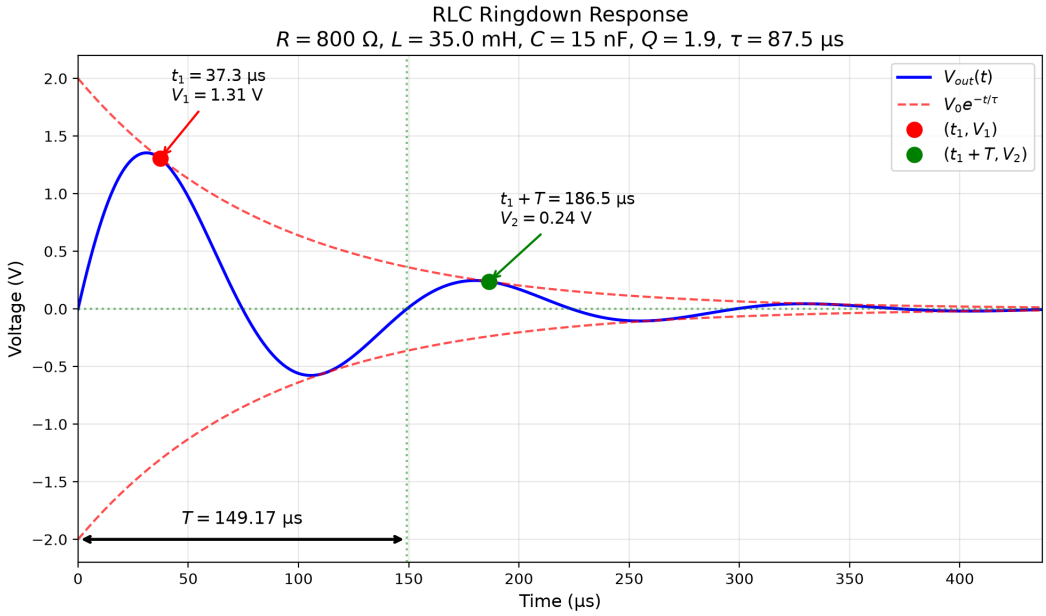


Figure L3.3: RLC ringdown after a step-function voltage drop. A peak (t_1, V_1), the point (t_1+T, V_2) one period later, and the period T are marked. The red dashed lines show the exponential envelope $V_0\, e^{-t/\tau}$.

Internal resistance

Just as in Lab 2, the wave generator has an internal resistance of about $600\,\Omega$, which is significant compared to the resistance R_R of the resistor and R_L of the inductor. The value of R in the RLC circuit is the sum of all three:

$$R = R_R + R_L + R_{\text{int}}. \tag{L3.9}$$

L3.3 Materials

- Multimeter
- Oscilloscope (10 MHz Picoscope 2204A with accompanying software) and two probes
- Breadboard
- Capacitor (rated 15 nF)
- Resistors (rated 100 Ω and 100 kΩ)
- Inductor (unknown value)
- Wires

L3.4 Instructions

RLC response curve

- Use the multimeter to measure the exact resistance of the resistor rated 100 Ω and of the inductor. Note the minimum resolution of the multimeter reading: this gives an estimate of the error in the measured resistance.

Resistance measurements

Resistor resistance (R_R): _____ ± _____ Ω
(measured with multimeter)

Inductor resistance (R_L): _____ ± _____ Ω
(measured with multimeter)

- Set up the series RLC band-pass circuit of the Background section as shown in Figure L3.4.

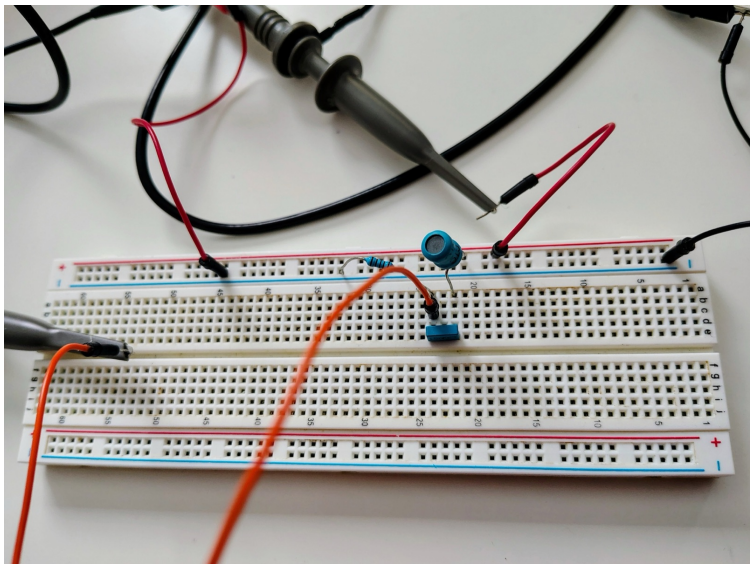


Figure L3.4: The series RLC band-pass circuit on a breadboard.

- Using the Picoscope software, set the timescale on the scope to 100 µs/div, the maximum voltage scale to 500 mV, and the trigger to auto (you may need to adjust it manually later to get a stable signal).
- Set the waveform generator to the sine-wave setting and the amplitude to 2 V.

- Scan the frequency in 1 kHz steps from 2 kHz to 15 kHz. Use the built-in "peak-to-peak" measurement of the Picoscope software to measure V_{out} as a function of frequency. (Note that the amplitude of V_{out} is *half* the peak-to-peak voltage.)

  Record these readings in your notebook.

- Focus on the peak of V_{out}. Scan near the peak in 0.1 kHz steps to find f_0, and estimate the error of this measurement.

Resonance frequency

Resonance frequency (f_0): _____ ± _____ kHz
(peak of response curve)

- Do the same to find the frequencies f_{low} and f_{high} at which V_{out} is $1/\sqrt{2}$ of its maximum value. (Hint: use a horizontal ruler to find the right value of V_{out}.) Use these to find the bandwidth Δf.

Bandwidth measurements

Lower frequency (f_{low}): _____ ± _____ kHz
(at $1/\sqrt{2}$ of max amplitude)

Upper frequency (f_{high}): _____ ± _____ kHz
(at $1/\sqrt{2}$ of max amplitude)

Bandwidth (Δf): _____ ± _____ kHz
($f_{high} - f_{low}$)

RLC ringdown

- Set the Picoscope to $V_{\text{max}} = \pm 200$ mV and the timescale to 50 µs/div.
- Set the generator waveform to "square", the amplitude to 1.5 V and the frequency to 1 kHz.
- Use a horizontal ruler to mark $V = 0$, and vertical rulers to mark two places where the signal crosses zero. Use these to determine T, and estimate the error in that measurement.

Period measurement

Period (T): _____ ± _____ µs
(time between zero crossings)

- Use vertical and horizontal rulers to identify the points (t_1, V_1) and $(t_1 + T, V_2)$, as in Figure L3.3.

Amplitude decay measurements

First amplitude (V_1): _____ ± _____ mV
(peak amplitude at time t_1)

Second amplitude (V_2): _____ ± _____ mV
(peak amplitude at time $t_1 + T$)

RC high-pass response curve

- Set up the high-pass RC circuit of the Background section as shown in Figure L3.5. Use the resistor rated 100 kΩ.

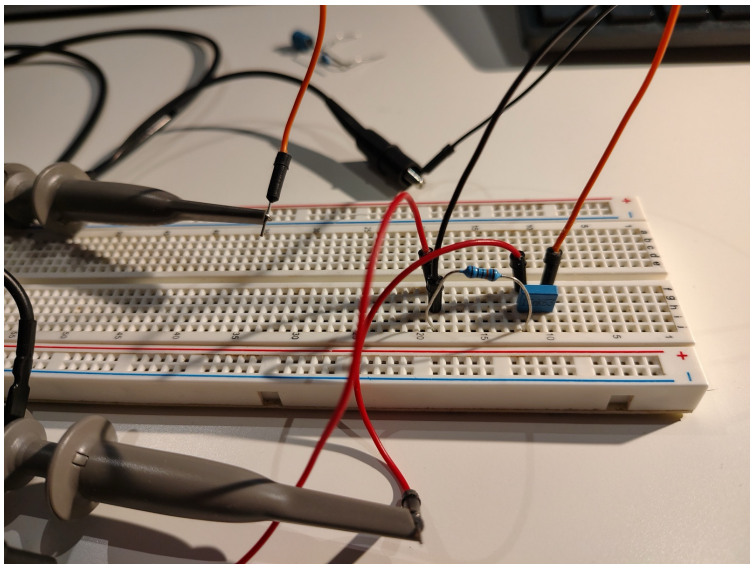


Figure L3.5: The series RC high-pass circuit on a breadboard (the same layout as in Lab 1).

- Use the multimeter to measure the exact value of R_R.

RC circuit resistance

Resistor resistance (R_R): _____ ± _____ kΩ
(measured with multimeter)

- Set the oscilloscope to $V_{\text{max}} = \pm 1$ V and the timescale to 2 ms/div.
- Set the generator waveform to "sine" and the amplitude to 800 mV.
- Scan the frequency range from 20 Hz to 500 Hz, noting V_{out} with the peak-to-peak measurement tool.

  Record these readings in your notebook.

- Focus on the region where V_{out} is $1/\sqrt{2}$ of its maximum value, and estimate the cutoff frequency f_c.

Cutoff frequency

Cutoff frequency (f_c): _____ ± _____ Hz
(at $1/\sqrt{2}$ of max amplitude)

L3.5 Analysis and discussion

Show your derivations for all calculations. When computing an error for a derived quantity, use the rules in Error propagation in the Lab Guide.

- Plot the RLC response curve using your readings of V_{out} as a function of frequency. How does the curve qualitatively compare to the theoretical curve of Figure L3.2?

- Using the measured bandwidth Δf and resonant frequency f_0 of the RLC response curve, as well as the capacitance of the $\sim 15\,\text{nF}$ capacitor measured in Lab 1, compute the internal resistance of the Picoscope wave generator and the inductance of the inductor. Show your derivations and estimate your errors. Are the results what you would expect? Why or why not?
- Using the period and amplitudes measured from the RLC ringdown curve, as well as the measured capacitance of the capacitor, compute the internal resistance of the Picoscope wave generator and the inductance of the inductor. Show your derivations and estimate your errors. Compare these results to those from the response curve. Explain any differences.
- Plot the RC response curve using your readings of V_{out} as a function of frequency. How does the curve qualitatively compare to the theoretical curve of Figure L3.1?
- Using the measured cutoff frequency f_c and the measured resistance of the resistor, determine the capacitance. Show your derivations and estimate your errors. Compare this with the value of the capacitance measured in Lab 1. Comment on any differences.

  Bonus: estimate C using the slope of the response curve at low frequencies.

L3.6 Instructions for the lab report

Submit a lab report that includes:

- A short *Introduction* giving background and context (e.g. explaining what filters are and how they are used) and explaining the goals of the experiment.
- A *Results* section giving measured values, tables and plots.
- An *Analysis* section showing how inferred (i.e. calculated) values are derived from measured ones.
- A *Discussion* section commenting on sources of error and comparisons between different measurements.
- A brief *Conclusions* section summarizing the results and placing them in context.